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Nonlinear Buckling Analysis and Ultimate Strength Prediction of Stiffened Steel and Aluminium Panels

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ABSTRACT

A computational model for buckling and postbuckling analysis of stiffened panels is developed. The model provides fast and accurate results for use in design of ships and offshore structures. The loads considered are in-plane compression or tension, shear force, and lateral pressure. Deflections are represented by trigonometric functions, and the principle of minimum potential energy is applied. Geometrical nonlinearities are accounted for using large deflection plate theory. Material nonlinearity is not taken into account, since the onset of yielding is taken as the capacity limit. Various computations have been performed for verification of the proposed model, and comparisons are made with nonlinear finite element methods.

INTRODUCTION

Ships and offshore structures are built up of stiffened plates. The buckling strength of each panel is vital for the overall strength of the structure. The postbuckling behavior is especially interesting because of the reserve strength after initial buckling due to development of membrane strains. Permanent deformation is unwanted, and the onset of membrane stress yielding is therefore used as a limiting design criterion. This means that material nonlinearities are not accounted for, which gives a large computational advantage.

In recent years, the use of computational tools have become common in design, and the need for explicit design formulas have therefore decreased. More direct and accurate calculations may be performed in order to achieve safe and optimal design. This has also been recognized by the ship Classification Societies, e.g. Det Norske Veritas (Steen 2001), where a new buckling

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standard based on direct calculations is being implemented. Nonlinear finite element analysis is not practical for design purposes due to the cost of modeling and computing. The method presented here is intended to provide accurate results with a minimum of modeling effort and computational cost. Recent work in the field of linear buckling of stiffened plates are for instance due to Hughes (1996) , while the nonlinear buckling response of unstiffened plates was investigated by Ueda (1987). In the current work, nonlinear plate theory is applied both for the plate and the stiffeners in a complete stiffened panel.

A typical stiffened panel consists of a rectangular plate field with longitudinal stiffeners in one direction and heavy transverse girders in the other direction, as shown in Fig. 1. This configuration is representative for the deck, side or bottom of a ship hull girder. The loads considered are in-plane compression or tension, shear force, and lateral pressure.

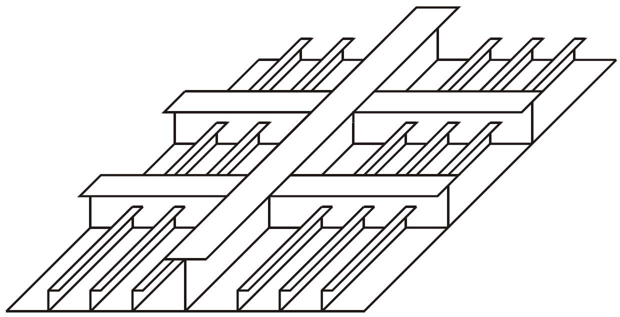


Fig. 1. Typical stiffened panel

THEORY

The governing equations are established using the principle of minimum potential energy:

$$\delta\Pi = \delta U + \delta T \tag{1}$$

where Π is total potential energy and δ is the variational operator. The internal strain energy U and the potential of external loads T are given as

$$\delta U = \int_V \sigma_i \delta\varepsilon_i dV \quad , \quad \delta T = \int_A t_i \delta u_i dA \tag{2}$$

Representing the displacement by a series of geometric functions with the amplitudes A_{mn}, the principle of minimum potential energy is reformulated using the Rayleigh-Ritz method. An incremental procedure is applied in order to linearize the equations. Arc-length incrementation is used, so that complex responses like snap-through may be analyzed. The Love-Kirchoff assumption is applied, which means that the strain in a material point is written as the sum of a bending part and a membrane part:

$$\varepsilon_{ij}^{tot} = \varepsilon_{ij} - zw_{,ij} \tag{3}$$

where z is the vertical distance from the plate neutral axis to the material point in consideration, and w is the plate deflection. The membrane strains are calculated according to Marguerre's plate theory (Marguerre 1937):

$$\begin{aligned} \varepsilon_x &= u_{,x} + \frac{1}{2} w_{,x}^2 + w_{0,x} w_{,x} \\ \varepsilon_y &= v_{,y} + \frac{1}{2} w_{,y}^2 + w_{0,y} w_{,y} \\ \gamma_{xy} &= u_{,y} + v_{,x} + w_{,x} w_{,y} + w_{0,x} w_{,y} + w_{,x} w_{0,y} \end{aligned} \tag{4}$$

The requirement of stress equilibrium and strain continuity can be combined into one single equation using the Airy stress function F, which is defined so that $\sigma_x=F_{,yy}$, $\sigma_y=F_{,xx}$, and $\tau_{xy}=F_{,xy}$. By differentiation and combination of Marguerre's equations, application of Hook's law, and substitution of the stress function, the plate compatibility equation is obtained:

$$\nabla^4 F = E(w_{,xy}^2 - w_{,xx} w_{,yy} + 2 w_{0,xy} w_{,xy} - w_{0,xx} w_{,yy} - w_{,xx} w_{0,yy}) \tag{5}$$

LOCAL BUCKLING

Local buckling of a plate with a single stiffener is considered first. By local buckling, it is understood that the connection between the plate and the stiffener does not displace vertically. The plate deflection shape is taken as a combination of a simply supported mode and a clamped mode in the transverse direction, Fig. 2.

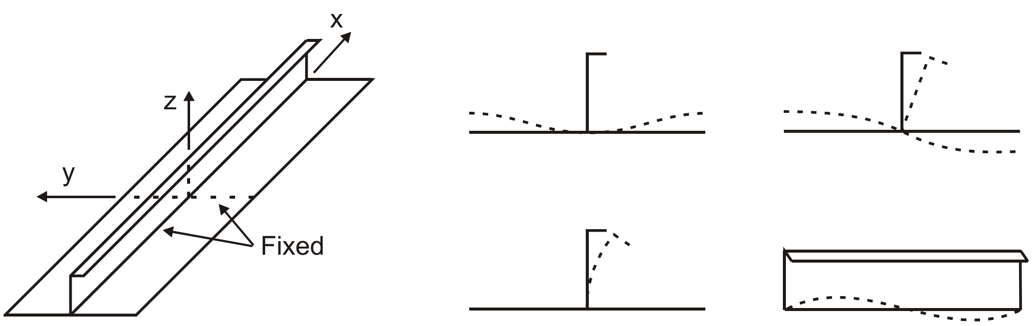


Fig. 2. Local deflection shapes

The edges of the plate are assumed to be free to move in-plane, but forced to remain straight. This restriction is imposed due to the effect of the neighboring plates that support the plate in a larger structure. The additional and initial deflections are taken as double Fourier series. The initial deflection represents the out-of-plane imperfection resulting from the fabrication process. The displacement function for the additional deflection is

$$w_1(x,y) = \sum_{m=1}^{Ms} \sum_{n=1}^{Ns} A_{mn}^{s} \, \mathbf{sin}(\frac{m\pi x}{a}) \mathbf{sin}(\frac{n\pi y}{b}) + \sum_{m=1}^{Mc} \sum_{n=1}^{Nc} \frac{A_{mn}^{c}}{2} \mathbf{sin}(\frac{m\pi x}{a})(1 - \mathbf{cos}(\frac{2n\pi y}{b})) \tag{6}$$

where a is plate length, and b plate width. The initial deflection is similar. The sine-terms represent the simply supported deflection mode, while the cosine-terms represent the clamped mode. No assumptions are made regarding the deflection amplitudes. The deflection shape adjusts itself to the one which minimizes the total energy of the system, depending on the plate geometry and loading.

A stress function for pure sine deflection without imperfections was derived by Levy (1937), and can be written as

$$F = -\frac{P_x y^2}{2bt} - \frac{P_y x^2}{2at} - \frac{P_{xy} xy}{t} + \sum_0^{2Ms} \sum_0^{2Ns} f_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \tag{7}$$

where the coefficients f_{mn} are second order functions of the displacement amplitudes, and P_x, P_y and P_{xy} are in-plane loads. A stress function for the combined sine/cosine deflection including initial deflection has been derived, and can be found in (Byklum 2002).

The displacement shape chosen for the stiffener consists of a rigid torsion part and a web bending part, Fig. 2. The displacement function is written as

$$v(x) = \frac{z}{h} \sum_{m=1}^{Ms} V_{1m} \sin\left(\frac{m\pi x}{a}\right) + \left(1 - \cos\left(\frac{\pi z}{2h}\right)\right) \sum_{m=1}^{Ms} V_{2m} \sin\left(\frac{m\pi x}{a}\right) \tag{8}$$

where h is the stiffener web height. The initial stiffener deflection is similar. Restrictions on the stiffener deflection are imposed by requiring continuity of rotation between the plate and the stiffener web:

$$\left.\frac{\partial v}{\partial z}\right|_{z=0} = -\left.\frac{\partial w}{\partial y}\right|_{y=0} \tag{9}$$

Longitudinal continuity is ensured by requiring equal displacement in the longitudinal direction for the plate and the stiffener:

$$\int_a u^p_{,x} dx = \int_a u^s_{,x} dx \tag{10}$$

The stiffener membrane strain is calculated by assuming that the longitudinal displacement is constant over the height of the stiffener, so that:

$$u^s_{,x} = \varepsilon^s_x - \frac{1}{2} v^2_{,x} - v_{,x} v_{0,x} = \text{constant} \tag{11}$$

Having computed all strain components in the plate and the stiffener, the potential energy is calculated by analytical integration over the plate and stiffener volume, using Eq. 2. Closed form expressions are obtained for all contributions. Using an incremental form of the principle of minimum potential energy, the nonlinear response is traced by incrementation using the equilibrium equation

$$\mathbf{K}\dot{\mathbf{A}} + \mathbf{G}\dot{\Lambda} = \mathbf{0} \tag{12}$$

where Λ is a general load parameter. The incremental stiffness matrix **K** and incremental load vector **G** are calculated by differentiation:

$$K_{mn,pq} = \frac{\partial^2 \Pi}{\partial A_{mn} \partial A_{pq}} \quad , \quad G_{mn} = \frac{\partial^2 \Pi}{\partial A_{mn} \partial \Lambda} \tag{13}$$

GLOBAL BUCKLING

The local buckling model is extended to a complete stiffened panel including several stiffeners. The local deflection is assumed to be the same for all stiffeners in the panel. This means that potential energy for one stiffened plate may be multiplied to represent the whole panel.

The global deflection is then included in the model. In order to limit the number of unknowns in the model, it is assumed that one single global degree of freedom is sufficient to represent the global deflection. Hence, the deformation shape is predefined, so that only the amplitude of the deflection changes during the analysis. The linear global eigenmode is used as an approximation for the nonlinear deformation shape. The global deflection shape is

$$w_g(x,y) = A_g \sum_{m=1}^{M_g} \sum_{n=1}^{N_g} k_{mn} \, \mathbf{sin}\left(\frac{m\pi x}{a}\right) \mathbf{sin}\left(\frac{n\pi y}{B}\right) \tag{14}$$

and similar for the initial global deflection. B is the width of the whole panel, in contrast to b which is the stiffener spacing. The amplitude A_g is the global degree of freedom, while the coefficients k_{mn} determines the deflection shape. The additional potential energy due to the global deflection causes a coupling between the local and the global deflections, which gives coupling terms in the total stiffness matrix.

Verification of the combined local and global buckling model is done by comparison with finite element analysis on two quite different stiffened panels, Fig. 3 and Table 1. The first is a typical steel panel, which is quite stocky. It is built up of three angle profiles. The other is a much more slender aluminium panel, built up of three tee-profiles. Imperfections are taken according to DNV Classification Note (1995).

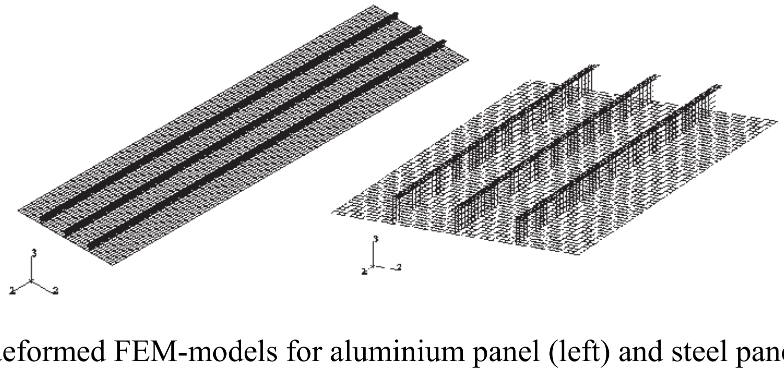
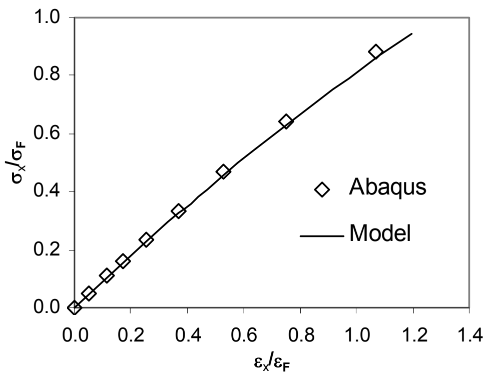


Fig.3. Undeformed FEM-models for aluminium panel (left) and steel panel (right)

Table 1. Geometry of stiffened panels used for verification

Panel	*a [m]*	*b[m]*	*B [m]*	*t [mm]*	*h [mm]*	*tw [mm]*	*bf [mm]*	*tf [mm]*	σ_F *[MPa]*	*E [MPa]*
Steel	2.73	0.85	3.40	16.5	350	12.0	100	17.0	355	208000
Alum.	2.40	0.32	1.28	5.0	75	5.0	40	5.0	240	70000

The finite element analyses are performed with the nonlinear code ABAQUS (Hibbit 1994). The panels are modeled using 4-node double curvature general-purpose shell elements. One panel breadth is modeled in the transverse direction, and (½ + 1 + ½) panel lengths in the longitudinal direction. Simply supported boundary conditions are applied on the longitudinal edges, while symmetry conditions are applied on the transverse edges. All edges are kept straight. Results for the two panels using the presented model and ABAQUS are given in Fig. 4 and Fig. 5. The load-end shortening are presented for axial load and transverse load for each panel. It is seen that the correspondence between the model and FEM is good.



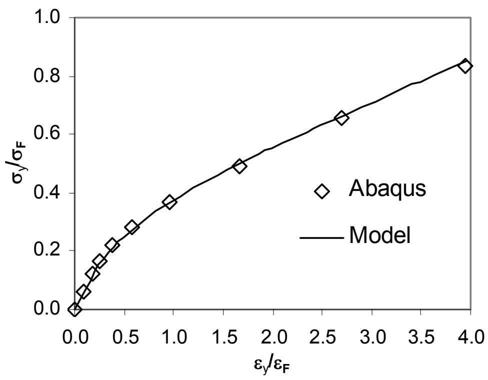
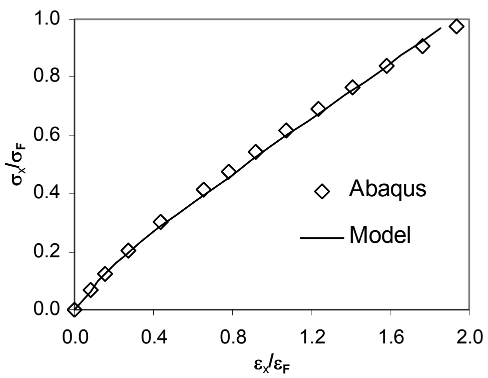


Fig.4. Normalized load-end shortening curves for steel panel under axial loading (left) and transverse loading (right)



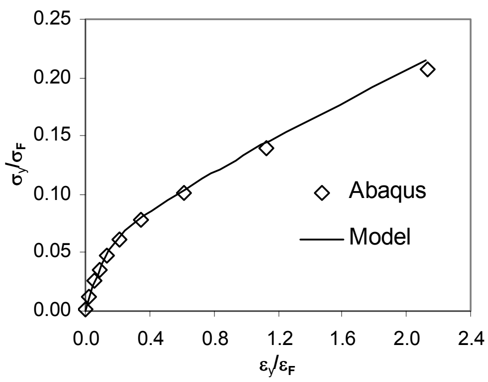


Fig.5. Normalized load-end shortening curves for aluminium panel under axial loading (left) and transverse loading (right)

Even though the shapes of the response curves are quite similar for the steel and the aluminium panel, the actual deflection shapes are very different. The steel stiffeners are strong, and the deflection shape is almost purely local. The aluminium stiffeners are weak, and deflection in the global mode is dominating. This shows that the model is well suited both for stocky and slender stiffeners.

DETERMINATION OF DESIGN CAPACITY

A first yield approach is used for estimation of the collapse load using the proposed buckling model. For each increment in the analysis, the membrane stress in certain critical points are checked using the von Mises yield criterion. For aluminum panels, the effect of reduced yield stress in the heat affected zones can be treated in a simplified manner by reducing the yield stress in the HAZ-zones. Residual stresses can be accounted for by adding the initial stress to the stress resulting from the buckling deflection.

An example of design capacities obtained for biaxial loading for the steel and the aluminium panel is given in Fig. 6. The loads are determined by checking the membrane stress for yielding at the connection between the plate and the web. The FEM-analysis are performed with elastic-plastic material without hardening. The intention is to provide design loads that are reasonably accurate and conservative. The results should therefore always be on the safe side, and not necessarily as close as possible to the real collapse load. In this respect it is considered that the present approach is successful for the two panels studied. It is seen that the difference between the design estimates and the ABAQUS collapse loads are larger for the steel panel than for the aluminium panel. This is because the former has a larger reserve capacity after the onset of yielding, probably due to its more stocky geometry. Results using the DNV Classification Note 30.1 (1995) are also included for comparison. The CN30.1 results are quite unconservative for the slender panel. It seems that the strength reduction due to load interaction in the biaxial region is larger for the model calculations than for the CN30.1 interaction curve.

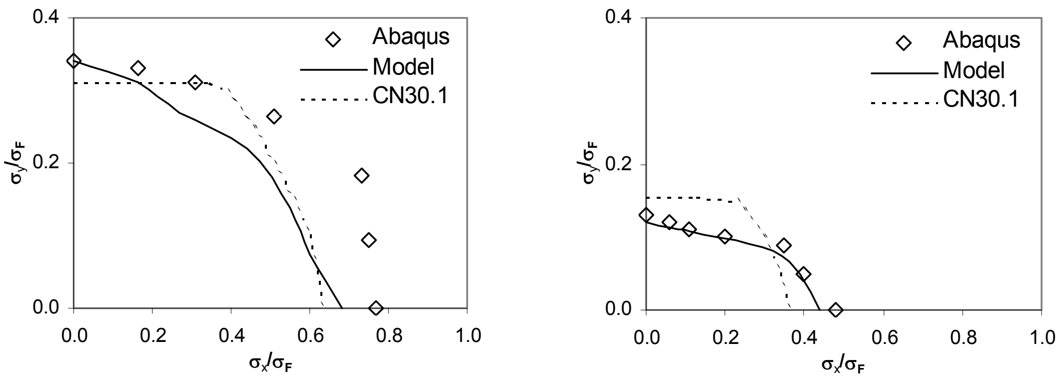


Fig.6. Interaction curves for biaxial loading for steel panel (left) and aluminium panel (right)

DISCUSSION AND CONCLUSIONS

A direct calculation model for buckling and postbuckling analysis of stiffened panels has been developed. All the load conditions that are typical for ships and offshore structures are included in the model, although only a few selected cases have been presented. Energy principles are applied, and geometrical nonlinearities are accounted for using large deflection plate theory.

The most important advantage of the method compared to nonlinear finite element methods is a large gain in computational efficiency. In addition, no geometric modeling is necessary, and no mesh has to be created. Finally, the input of imperfections is very easy, since the user has full control of the initial deflections. Compared to conventional design formulas, the major advantage of the method is the more direct calculation strategy which gives increased accuracy. This is especially important for non-standard geometries that the explicit design formulas were not originally created for. More information is also obtained in form of displacement shapes and amplitudes. Analyses may be performed with different imperfections, in contrast to the design formulas where a fixed imperfection is implicit in the expressions.

Computations have been performed on a variety of plate and stiffener geometries for verification of the proposed model, and comparisons are made with nonlinear finite element methods. Some examples are presented. The load-deflection curves correspond well with the results from the finite element analyses. Design resistance is obtained by a first yield criterion. The estimates are reasonable and conservative compared to the collapse loads obtained from finite element analysis. Further work will be performed in order to determine which collapse criterions are most appropriate for different load conditions.

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