International Conference on Computational Methods in Marine Engineering
MARINE 2009
T. Kvamsdal, B. Pettersen, P. Bergan, E. Oñate and J. García (Eds)

APPROXIMATE POSTBUCKLING ANALYSIS OF IMPERFECT STIFFENED PLATES WITH A FREE EDGE

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Key words: Stiffened plates; Free edge; Postbuckling analysis; Semi-analytical method.

Summary. A large deflection, semi-analytical method for pre- and postbuckling analysis of stiffened plates with a free edge is presented. The formulations derived are implemented into a FORTRAN computer program, and numerical results are obtained for a variety of plate and stiffener geometries. Load-deflection curves computed by the present model are compared with fully nonlinear finite element analyses. Good agreement is obtained.

1 INTRODUCTION

A semi-analytical method is presented that is a computationally efficient tool for pre- and postbuckling behaviour of stiffened plates with a free or partially free edge. In combination with a strength criterion, the method is able to predict the ultimate strength limit. Nonlinear finite element method analyses could be used for such predictions. However, such analyses are often too time consuming to prepare, run and post-process. Furthermore, traditional explicit design formulas are relatively simple to use, but they are normally not applicable to stiffened plates with a free edge.

2 SUMMARY OF THEORY

The rectangular plate considered can be defined with reference to Fig. 1. The plate has one unsupported edge that is free or provided with an edge stiffener, and it is supported in the out-of-plane direction at the three other edges. Two opposite supported edges, perpendicular to the free edge, are subjected to an external stress S_x. The assumed displacement field are defined by

$$w = w^a + w^b, \quad u = u^a + u^b + u^c, \quad v = v^a + v^b + v^c \tag{1}$$

where u, v and w represent the displacements in x-, y- and z-direction, respectively, and

$$w^a(x,y) = \sum_{i=1}^{M_{wa}} w_i^a \frac{y}{b}\sin(\frac{\pi i x}{L});\ w^b(x,y) = \sum_{i=1}^{M_{wb}}\sum_{j=1}^{N_{wb}} w_{ij}^b \sin(\frac{\pi i x}{L})\sin(\frac{\pi j y}{b}) \tag{2}$$

$$u^a(x,y) = \sum_{i=1}^{M_{ua}} u_i^a \frac{y}{b}\sin(\frac{\pi i x}{L});\ u^b(x,y) = \sum_{i=1}^{M_{ub}}\sum_{j=1}^{N_{ub}} u_{ij}^b \sin(\frac{\pi i x}{L})\sin(\frac{\pi j y}{b});\ u^c(x,y) = u^c \frac{x}{L} \tag{3}$$

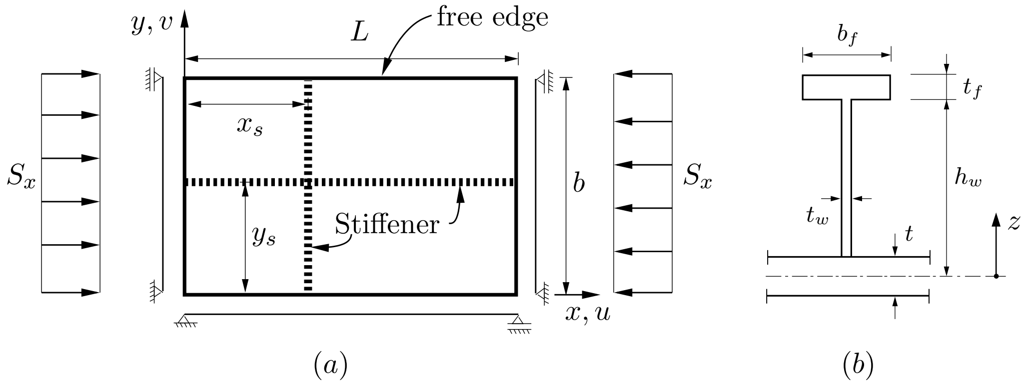

Figure 1: (*a*) A uniaxially loaded, stiffened plate with a free edge and three supported edges and (*b*) an eccentric stiffener.

$$v^a(x,y) = \sum_{i=1}^{M_{va}} v_i^a \frac{y}{b}\cos(\frac{\pi i x}{L});\ v^b(x,y) = \sum_{i=1}^{M_{vb}}\sum_{j=1}^{N_{vb}} v_{ij}^b \sin(\frac{\pi i x}{L})\sin(\frac{\pi j y}{b});\ v^c(x,y) = v^c\frac{y}{b} \quad (4)$$

where w_i^a, w_{ij}^b, u_i^a, u_{ij}^b, u^c, v_i^a, v_{ij}^b, v^c are amplitudes, L the plate length and b the plate width.

By using a solution procedure discussed in detail elsewhere (Steen [1], Brubak and Hellesland [2], and Steen, Byklum and Hellesland [3]), the model is able to trace the pre- and postbuckling response, and consequently, it accounts for a possible reserve strength beyond the elastic buckling load typical for slender plates. In this procedure, the equilibrium equations are solved incrementally by computing the rate form of the equilibrium equations with respect to an arc length parameter. In the common matrix notation, the final set of equations can be given by

$$\mathbf{K}\dot{\mathbf{d}} + \mathbf{G}\dot{\Lambda} = 0 \quad \text{and} \quad \dot{\Lambda}^2 + \frac{1}{t^2}\dot{\mathbf{d}}^T\dot{\mathbf{d}} = 1 \quad (5)$$

where, $\mathbf{K}$ is a generalised, incremental (tangential) stiffness matrix, $-\mathbf{G}\dot{\Lambda}$ is a generalised, incremental load vector, $\mathbf{d}$ is the displacement amplitudes and Λ is the load factor. The incremental stiffness matrix, load vector and displacement vector can be divided into submatrices and subvectors as given by

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{uv} & \mathbf{K}_{uw} \\ \mathbf{K}_{vu} & \mathbf{K}_{vv} & \mathbf{K}_{vw} \\ \mathbf{K}_{wu} & \mathbf{K}_{wv} & \mathbf{K}_{ww} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{G}_u \\ \mathbf{G}_v \\ \mathbf{G}_w \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{bmatrix} \quad (6)$$

where the displacement amplitude vector $\mathbf{d}$ is an assembly of all the displacement amplitudes. It can be written as

$$\mathbf{d}^T = [d_i] = \Big[u_1^a, ..., u_{M_{ua}}^a, u_{11}^b, u_{12}^b, ..., u_{M_{ub}N_{ub}}^b, u^c, v_1^a, ..., v_{M_{va}}^a, v_{11}^b, \\ v_{12}^b, ..., v_{M_{vb}N_{vb}}^b, v^c, w_1^a, ..., w_{M_{va}}^a, w_{11}^b, w_{12}^b, ..., w_{M_{wb}N_{wb}}^b\Big] \quad (7)$$

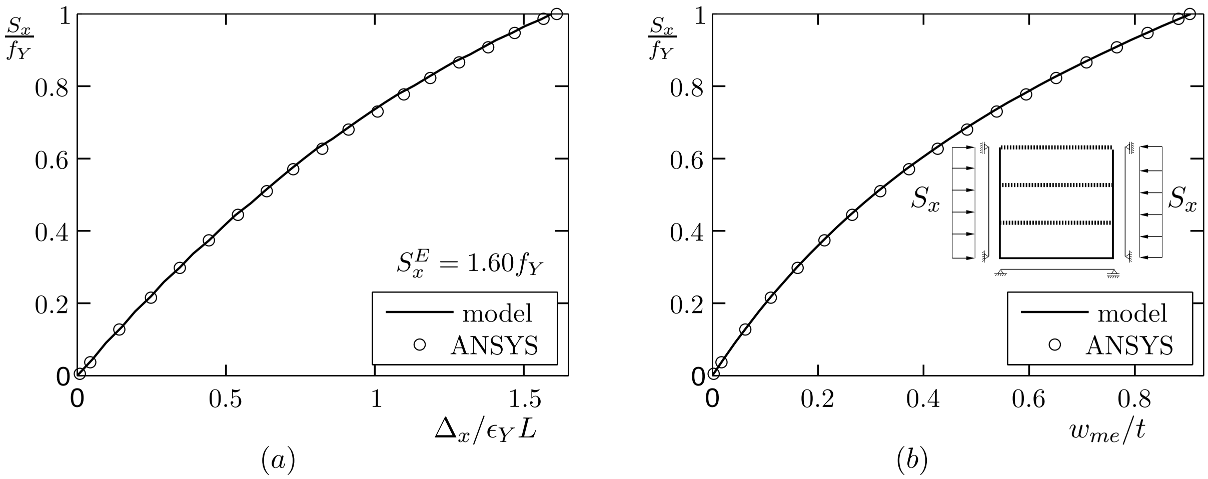

Figure 2: (a) Load-shortening and (b) load-deflection curves of a plate ($L/b/t$ = 1000/1000/12 mm) provided with three flat bar stiffeners (h/t_w = /1000/10 mm) and subjected to a uniaxial load S_x.

When the solution of Eq. 5 at a stage "k" is computed, the displacement amplitudes and load parameter at the next stage "$k+1$" are then obtained from a linear Taylor series expansion as

$$d_j^{k+1} = d_j^{\,k} + \dot{d}_j^{\,k} \Delta\eta; \qquad \Lambda^{k+1} = \Lambda^k + \dot{\Lambda}^k \Delta\eta \tag{8}$$

In this manner, the solution propagation is continued until a specified limit, or a given strength criterion, is reached. The present solution procedure is capable of passing limit points, including tracing of snap-through and snap-back equilibrium curves. A more detailed presentation of the method is given by Brubak [4].

3 LOAD-DISPLACEMENT RESULTS

Elastic load-displacement results obtained by the present model have been compared with finite element analysis results (ANSYS [5]; shell elements Shell93 both for plate and stiffeners) for a variety plate and stiffener dimensions. For verification purposes, the imperfection shape is taken equal to the first buckling mode with a maximum out-of-plane value $w_{0,max} = 5$ mm. The elastic material properties are Young's modulus $E = 208000$ MPa and Poisson's ratio $\nu = 0.3$.

Typical load-deflection results are shown in Fig. 2 for the stiffened plate shown by the insert in the figure. Both load-shortening and load-deflection curves are presented. The results are given in a non-dimensional form. In the figure, Δ_x is the end shortening, $f_Y = 235$ MPa is the yield strength, $\epsilon_Y = f_Y/E$ $(= 0.00113)$ is the yield strain and w_{me} is the additional out-of-plane displacement at the midlength of the free edge. The agreement between the response curves computed by the present model (thick solid curves) and by ANSYS (open dots) is good. Similar agreement is also achieved for other plate and stiffener dimensions. In the figure, $S_x^E = 1.60 f_Y$ is the elastic buckling stress (eigenvalue).

4 BUCKLING STRESS AND ULTIMATE STRENGTH RESULTS

Elastic buckling stress limit (ESL) results computed by the present model and by ANSYS have also been compared, and the agreement between the corresponding results is very good. These ESL results give an indication on the slenderness of the plates.

Furthermore, calculated ultimate strength limit (USL) results using first yield of the von Mises membrane stress as strength criterion [6] have been examined (Brubak [4]) for selected unstiffened plates with a free edge. Results, not included here, are compared to fully nonlinear element analysis results obtained using ANSYS. Additional work remains to be done.

5 CONCLUDING REMARKS

An efficient computational model is presented for large deflection postbuckling analysis of imperfect, stiffened plates with an edge being free or provided with an edge stiffener. The applicability of the present method is documented for several cases by comparison with finite element analysis results using ANSYS.

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