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Approximate buckling strength analysis of plates with arbitrarily oriented stiffeners

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Summary Buckling of stiffened plates with arbitrarily oriented stiffeners are considered. The main objective of the work has been to develop a computational model for direct calculation of the structural response using a semi-analytical method. The deflections are represented by trigonometric functions. All combinations of in-plane shear, and biaxial in-plane compression or tension are included in the formulations. Estimation of the buckling strength is made using first yield as the strength criterion. The formulations derived are implemented in a Fortran computer code. Numerical results are obtained for a variety of stiffener orientations and geometries. The results are compared to finite element analysis results and are found, in most cases, to be conservative compared to the finite element calculation results.

Introduction

Stiffened plates are used extensively in ships, aircrafts, bridges and offshore installations. Traditionally, explicit design formulas [1, 6] have been used to provide quick strength estimates of stiffened plates. These formulas are relatively simple to use, but they are normally not very applicable with respect to arbitrary orientations of the stiffeners.

This paper presents a semi-analytical model 1) for calculation of the elastic buckling load (eigenvalue) of stiffened, simply supported plates with in-plane loading and arbitrarily oriented stiffeners, and 2) for a conservative buckling load assessment of such plates with specified imperfections using the first yield as the strength criterion. The present method provides relatively high numerical accuracy with low computational effort. As an alternative, the finite element method could have been used, but this method is still unpractical and too time consuming for most design purposes at present.

The stiffeners are assumed to be sniped at their ends and only their out-of plane bending (beam) stiffness are included in strain energy. Thus their axial stiffness and its influence on the internal membrane stress distribution is neglected.

Elastic buckling limit state (ELS)

Ideal elastic buckling loads (eigenvalues) of a perfectly plane plate are computed using the Rayleigh-Ritz method for the plate in Fig. 1. It is simply supported and subjected to in-plane shear and biaxial compression or tension. It may have several arbitrarily oriented stiffeners, but only one is shown in the figure. The assumed displacement field, which satisfies the boundary conditions, is given by

$$w(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\sin(\frac{\pi i x}{L})\sin(\frac{\pi j y}{b}) \qquad \text{where} \quad \begin{array}{lcl} 0 & \leq x \leq & L \\ 0 & \leq y \leq & b \end{array} \tag{1}$$

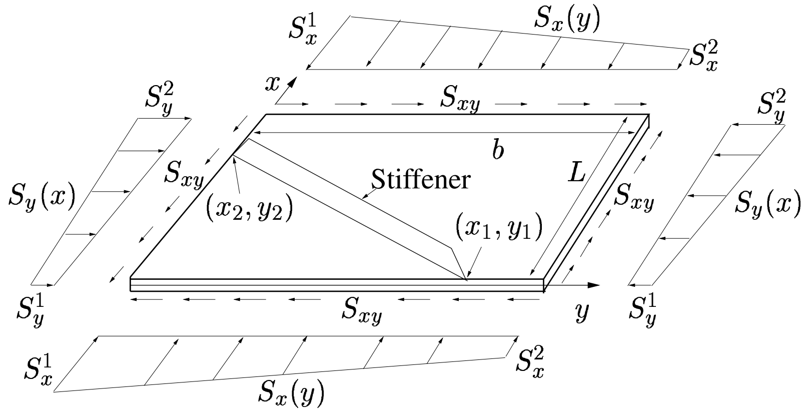


Figure 1: Simply supported plate with arbitrarily oriented stiffeners. The coordinates of the ends of the stiffener are (x_1, y_1) and (x_2, y_2). S_x and S_y are the applied stress in x- and y-direction respectively. The plate thickness is t_p.

where a_{ij} are amplitudes, L is the plate length and b is the plate width. The first step is now to establish the potential energy of the plate, $\Pi = U + T$, where U is the strain energy and T is the potential energy of the external loads. Equilibrium requires that Π has a stationary value. This requirement leads to displacements that must satisfy the matrix equation

$$(K^M_{ijkl} + \Lambda^e K^G_{ijkl}) da^e_{kl} = 0 \quad \text{, where} \quad K^M_{ijkl} = \frac{\partial^2 U}{\partial a_{ij} \partial a_{kl}} \text{ and } K^G_{ijkl} = \frac{\partial^2 T}{\partial a_{ij} \partial a_{kl}} \tag{2}$$

Here, K^M_{ijkl} is the material stiffness matrix, K^G_{ijkl} the geometrical stiffness matrix, Λ^e the eigenvalues and da^e_{kl} the eigenvectors.

Based on the Love-Kirchoff theory [2], the elastic strain energy contribution from bending of the plate can be given by

$$U^b_{\text{plate}} = \int_0^L \int_0^b \frac{Et^3}{24(1-\nu^2)} [(w_{,xx} + w_{,yy})^2 - 2(1-\nu)(w_{,xx} w_{,yy} - w^2_{,xy})] \, dx \, dy \tag{3}$$

The membrane strain energy is not included as it does not affect computed eigenvalues.

The displacements of the stiffeners are equal to the displacements in the plate along the stiffener. In the present model, the stiffeners are accounted for by using ordinary beam theory. This gives a bending strain energy defined by

$$U^b_{\text{stiffener}} = \frac{1}{2} \int_S EI_e \left(\frac{(x_2 - x_1)^2 w_{,xx} + 2(x_2 - x_1)(y_2 - y_1) w_{,xy} + (y_2 - y_1)^2 w_{,yy}}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right)^2 dS \tag{4}$$

where S is the length and I_e the effective moment of inertia of the stiffener. As shown in Fig. 1, the coordinate of the ends of the stiffener are (x_1, y_1) and (x_2, y_2). In general, this integral can not be solved analytically and numerical integration must be applied. The trapezoidal rule [4] is used in the present work.

By using Marguerre's plate theory [5], the potential energy of external loads due to plate bending becomes

$$T = -\frac{t}{2} \int_0^L \int_0^b (S_x(y) w^2_{,x} + 2S_{xy} w_{,x} w_{,y} + S_y(x) w^2_{,y}) \, dy \, dx \tag{5}$$

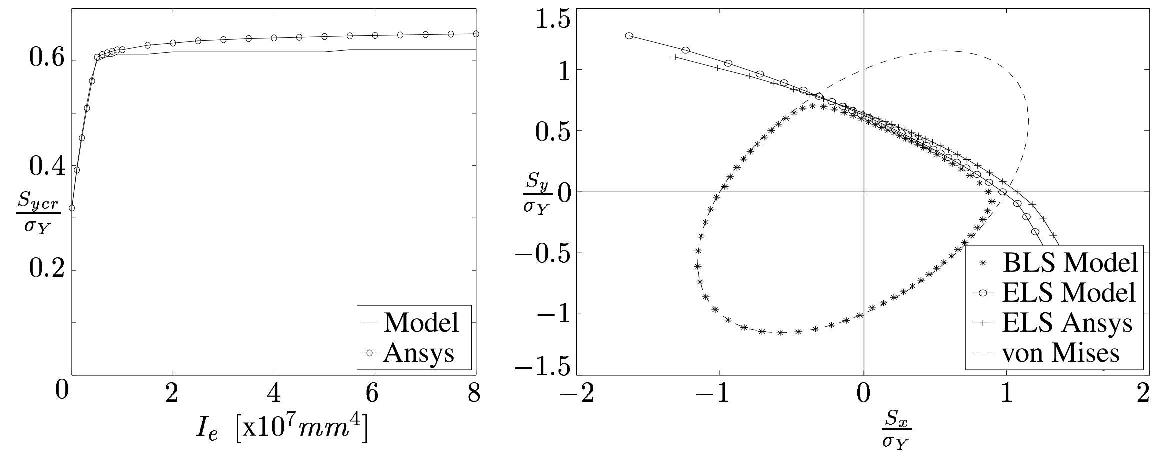


Figure 2: Left: The elastic (membrane) buckling stress of the plate subjected to constant external stress S_y versus I_e. Right: Biaxial loading interaction curves from different analysis for the plate with I_e = 4x$10^7 mm^4$

Buckling strength limit state (BLS)

In linearized elastic second order theory, the displacement w beyond the initial displacement imperfection (w_0) can be estimated from

$$w = \frac{\Lambda}{\Lambda^{cr} - \Lambda} w_0 \tag{6}$$

where Λ^{cr} is the eigenvalue and Λ the current load factor. This expression is used in this paper, for increasing load factors, to estimate a buckling strength (capacity) using first yield of the von Mises' membrane stress as the strength criterion. Use of Eq. 6 implies that this strength estimate will never exceed the elastic buckling load. The model is consequently conservative as it is not able to capture the reserve strength beyond this load.

In the present work, w_0 is the first eigenmode of the eigenvalue problem defined in Eq. 2. The membrane stresses in the plate is calculated by making use of Airy's stress function $F(x, y)$ which satisfies

$$\sigma_x^m = F_{,yy} \qquad \sigma_y^m = F_{,xx} \qquad \tau_{xy}^m = -F_{,xy} \tag{7}$$

where σ_x^m, σ_y^m and τ_{xy}^m are membrane stresses. Airy's stress function is found by solving the non-linear compatibility equation

$$\nabla^4 F = E(w_{,xw}^2 - w_{,xx}w_{,yy} + 2w_{0,xy}w_{,xy} - w_{0,xx}w_{,yy} - w_{0,yy}w_{,xx}) \tag{8}$$

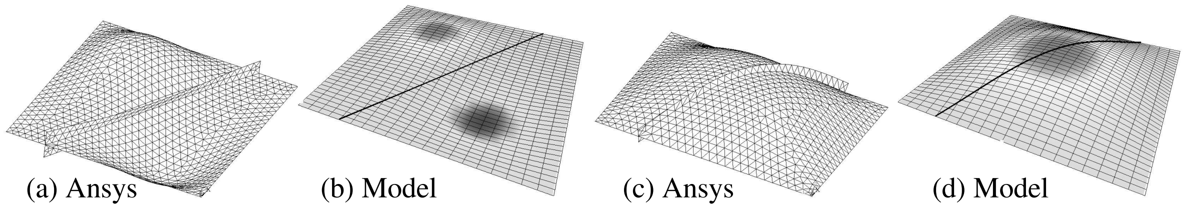
This equation is solved by Byklum [3] with the assumed displacement field w and an initial imperfection w_0.

Results

Elastic buckling stresses computed by the present model have been compared to finite element analysis results using Ansys for a variety of plate dimensions and stiffener orientations. Typical results of a plate with one inclined stiffener with various moments of inertia I_e, and with data otherwise given in Table 1, are shown in Fig. 2 (left). The stiffener used is a flatbar of thickness $t_w = 20mm$ and is symmetric about the middle plane.

Table 1: The dimensions of the plate and the stiffener orientation.

Plate $[mm]$			Stiffener $[mm]$		Material properties		
L	b	t_p	(x_1, y_1)	(x_2, y_2)	$E[MPa]$	ν	$\sigma_Y[MPa]$
2000	2000	20	(0,400)	(2000,1600)	208000	0.3	235



(a) Ansys (b) Model (c) Ansys (d) Model

Figure 3: The buckling mode calculated in Ansys and by the model with an effective moment of inertia $I_e = 5\text{x}10^7 mm^4$ [(a) and (b)], and with an effective moment of inertia $I_e = 0.5\text{x}10^7 mm^4$ [(c) and (d)].

In Fig. 2 (right), additional comparisons of ELS results are presented for one specific I_e of the stiffener and various biaxial load combinations. The agreement is generally good. The present model is conservative in the region of interest (within the von Mises ellipse) for this case, and also for most other cases investigated.

Fig. 2 (right) also presents the von Mises yield ellipse ("squash yield state") and interaction curves for the present buckling strength limit state (BLS). The latter results are computed with a maximum imperfection w_0 of $1mm$. This value is rather small, but adequate for the purpose of the present illustration.

Fig. 3 shows the buckling modes calculated by Ansys and by the semi-analytic model of a plate with a strong and a weak stiffener. The agreement is seen to be good.

Concluding remarks

An efficient computational model for elastic buckling analysis of a plate with arbitrarily oriented stiffeners is presented. Good agreement with finite element analysis of the elastic buckling stress is achieved, and the results are in most cases conservative compared to finite element analysis results. The buckling strength of a plate with imperfections is estimated using first yield as strength criterion. The model does not account for the reserve strength after the elastic buckling stress is reached. To establish such a model, using large displacement plate theory and energy principles [3], further work is needed.

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